Course Name: ATS 302: Mathematical Applications in the Earth Sciences
Course Credits: 4 credits
Term: Winter 2020
Class Time: MWF, 2-2:50, Wilkinson 235
Recitation: Th, 2-3:50, Burt 128
Prerequisites: MTH 252 or equivalent. No co-requisites.

Instructor:
Nick Siler
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Office Hours – 2 hours per week; times chosen by doodle poll.

Course Description:
An introductory survey of mathematical applications in climate science, meteorology, oceanography, geology, and geophysics. Topics include conservation laws, harmonic motion, exponential growth/decay, linear approximations, numerical methods, waves, diffusion, fluid flow, systems of equations, inverse problems, and data analysis.

Textbook (optional):
Calculus, Early Transcendentals, by Briggs & Cochran.

Evaluation of Student Performance:
- **Homework (50%):** Homework, consisting of problems and/or writing, will be assigned every Friday except during the midterm week and dead week, and will be due the following Friday. Students are encouraged to work together, but must note the names of collaborators on each homework.
- **Midterm (20%):** A cumulative take-home exam assigned on the Friday of week 5 and due the following Friday. Students must work independently, but can consult any text or online resource.
- **Final (20%):** A cumulative take-home exam assigned on the Friday of week 9 and due on the Monday of finals week. Students must work independently, but can consult any text or online resource.
- **Class Participation (10%)**

Student Learning Outcomes
Upon completion of this course, students will be able to
- Articulate the physical and conceptual meaning of mathematical equations, operators, and methods that are important in the earth sciences.
- Translate earth-science problems into mathematical equations and find the analytic solution.
- Write code to solve and/or plot solutions to earth-science problems.
- Write clear, logical homework solutions using appropriate mathematical notation.

Statement Regarding Students with Disabilities:

Accommodations for students with disabilities are determined and approved by Disability Access Services (DAS). If you, as a student, believe you are eligible for accommodations but have not obtained approval please contact DAS immediately at 541-737-4098 or at http://ds.oregonstate.edu. DAS notifies students and faculty members of approved academic accommodations and coordinates implementation of those accommodations. While not required, students and faculty members are encouraged to discuss details of the implementation of individual accommodations.

Expectations for Student Conduct:
http://studentlife.oregonstate.edu/code

Course Content:

Week 1: The power of the math you already know
Readings (optional): Briggs & Cochran (Chapters 3-6)
Math Topics:

- Physically useful functions/equations
- Applications of derivatives, chain rule, and integrals (1D)

Earth Science Applications:
Exponents (growth/decay), logarithms (decibels, Richter scale), power laws (earthquake frequency), normal distributions/probability, periodicity, rates of change (absolute and fractional), work, maxima/minima.

Week 2: Introduction to ordinary differential equations (ODEs)
Math Topics:

- Initial-value and boundary-value problems
- General and particular solutions
- Solutions to common initial-value problems (simple harmonic motion, exponential growth/decay)
- Introduction to Sturm-Liouville theory, Fourier series

Earth Science Applications:
Static stability/instability of fluids, buoyancy oscillations in the ocean/atmosphere, setting the stage for waves and diffusion.

Week 3: Approximations of nature
Readings (optional): Briggs & Cochran (Chapters 8.3 & 9)
Math Topics:

- MacClaurin/Taylor series
- Linearization (first-order Taylor expansion)
- Discrete derivatives, finite-difference method

Earth Science Applications:
Linear approximations of non-linear equations (e.g., Stefan-Boltzmann [radiation], Clausius-Clapeyron [water vapor], gravity), climate sensitivity/feedback, numerical solutions to initial-value problems (e.g., global warming due to a time-varying radiative forcing).

Week 4: Conduction and diffusion
Math Topics:

- Introduction to partial derivatives (functions of space and time)

- Differential forms of conservation laws for mass, energy, and momentum
- Diffusion equation (heat, mass, and momentum)
- Higher-order discrete derivatives; simulating numerical diffusion
- Separation of variables: analytic solutions and physical interpretation

Earth Science Applications:

Fourier's law (conduction), Fick's law (molecular diffusion), Darcy's law (flow in permeable medium), momentum transport (via friction), energy transport by atmospheric eddies. Specific earth-science examples include a dispersing plume, well discharge, mountain erosion, and poleward energy transport by the atmosphere.

Week 5: Waves

Math Topics:

- Wave equation (transverse vs. longitudinal, physical interpretation)
- d'Alembert solution
- Separation of variables solution; normal modes
- Midterm review

Earth Science Applications:

Seismic waves (primary and secondary), acoustic waves, shallow-water gravity waves (e.g., tsunamis), vibrating strings, spectral analysis.

Week 6: Multivariable calculus

Readings (optional): Briggs & Cochran (Chapters 12.4-12.6, 11.1-11.4, 13)

Math Topics:

- Functions in 2D and 3D space
- Partial derivatives; multidimensional chain rule
- Multiple integration
- Vectors, vector addition, dot/cross products
- Directional derivatives and the gradient; introduction to the Del operator

Earth Science Applications:

Chain-rule representation of climate feedbacks, regional/global averages (e.g., surface temperature), topographic flow lines.

Week 7: Vector calculus

Readings (optional): Briggs & Cochran (Chapter 14)

Math Topics:

- Divergence and curl operators
- Line integrals along a curve: work (force), circulation (fluid flow), Green's theorem
- Conservative vector fields, irrotational flow, potential fields
- Line integrals across a curve: 2D divergence
- Non-divergent flow and streamlines
- Divergence theorem (3D)

Earth Science Applications:

Geopotential/work, vorticity, divergence, geostrophic flow, velocity potential, streamlines.

Week 8: Linear systems of equations

Math Topics:

- Matrices and matrix operations (addition, multiplication)
- Systems of equations; Gauss-Jordan elimination
- Linear dependence/independence, row space, column space, rank.
- Least-squares solution for an over-determined system

Earth Science Applications:
Multiple linear regression, statistical forecast models, inverse problems (e.g., spectroscopy)

Week 9: Linear transformations in 2D/3D space

Math Topics:

- Matrices as linear operators
- Determinants and their geometric interpretation
- Orthogonal transformations; rotation and reflection
- Eigenvectors, eigenvalues, and diagonalization
- Orthogonal diagonalization of symmetric matrices

Earth Science Applications:
Stress tensor, principal stresses/axes, coordinate transformations.

Week 10: Data analysis in the time/space domain

Math Topics:

- Statistics of discrete data: mean, variance, standard deviation, higher-order moments.
- Correlation and covariance
- Eigenvectors/values of the covariance matrices: empirical orthogonal functions and principal components
- Singular value decomposition (SVD)

Earth Science Applications:
Patterns/modes of variability in the earth/ocean/atmosphere, data compression, data filtering