Creating a Differentiated Mathematics Classroom

Recognizing different mathematical learning styles and adapting differentiated teaching strategies can facilitate student learning.

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Two popular books on instruction provide excellent examples of how mathematics teachers might thoughtfully begin their units of instruction. Classroom Instruction That Works (Marzano, Pickering, & Pollock, 2001) and How to Differentiate Instruction in Mixed-Ability Classrooms (Tomlinson, 2001) provide different but overlapping visions of good beginnings.

The differences between the approaches, however, are striking. As shown in Figure 1 (p. 75), Marzano (Version 1) frontloads key vocabulary terms and points directly to required standards, whereas Tomlinson (Version 2) highlights choices for learning division and directly inquires into students' prior knowledge.

But what if we were to combine the two approaches? The teacher would begin the unit with specific goals, informing the students that they would learn a number of skills, such as how to use division to calculate quickly and accurately or how to develop good explanations about how and why division works. To assess students' areas of interest, the teacher could have students rank the level of appeal of several real-life situations that require a knowledge of division. Students would also respond to several questions related to their prior knowledge. Version 3 in Figure 1 shows this combined approach, which blends goals or standards with student interest and prior knowledge, thus linking our drive toward clarity of instructional purpose with a deeper regard for student differences.

As writers, researchers, and educators, we became interested some time ago in helping mathematics teachers find the proper balance between unity of instructional purpose and models of differentiation. We created a series of workshops entitled “Mathematical Styles and Strategies” in which we explored three questions:

- Do different students possess different styles of mathematical learning?
- What is the relationship between the research on effective strategies and student styles of mathematical learning?
- What constitutes a fair and thoughtful assessment of student progress in mathematics, given students' personal differences when it comes to learning mathematics?

On the basis of our findings, we developed a model of instruction that combines the approaches suggested in Classroom Instruction That Works and in How to Differentiate Instruction in Mixed-Ability Classrooms. We call this hybrid “differentiation that works.”
What Is Mathematical Learning Style?

Differences in students’ mathematical learning styles emerge quite early in their development. In a remarkable series of interviews, Tang and Ginsberg (1999) asked four preschoolers to solve similar problems in counting, using coins and drawings to represent their thinking. Kenneth worked in a step-by-step manner. First, he separated his coins into one group of two and a second group of three; next, he pushed them together to form a straight line; finally, he rearranged the coins into a pattern and counted the result. In a later interview, he visualized and created images by turning his coins into parts of imaginary faces and counting “eyebrows and ears.” Robert and Pedro searched for patterns and categories by sorting their coins before or while they counted and by acknowledging the reasons behind their strategy. Omar established a personal relationship with the interviewer, physically aligning his hands with hers before counting off on his fingers.

The students’ responses indicate style-based dispositions, not Piagetian developmental levels. The responses fall into four style-based patterns familiar to those acquainted with the Myers-Briggs Type Indicator (Briggs & Myers, 1962/1998), our Learning Preference Inventory (Hanson & Silver, 1991), or our Learning Style Inventory (Silver & Strong, 2003). Our inventories characterize the four styles as follows:

- The Mastery style: People in this category tend to work step-by-step.
- The Understanding style: People in this category tend to search for patterns, categories, and reasons.
- The Interpersonal style: People in this category tend to learn through conversation and personal relationship and association.
- The Self-Expressive style: People in this category tend to visualize and create images and pursue multiple strategies.

Students can acquire and make use of more than one style, as Kenneth did. Although people can work in all four styles, we generally tend to develop preferences or strengths in one or two of the styles and develop weaknesses in the ones that remain.

In interviews we conducted with more than 200 students over the past seven years, we consistently found these patterns of thinking. For instance, we presented students with the following problem:

Nineteen campers are hiking through Acadia National Park when they come to a river. The river moves too rapidly for the campers to swim across it. The campers have one canoe, which holds three people. On each trip across the river, one of the three canoe riders must be an adult. There is only one adult among the 19 campers. How many trips across the river are necessary to get all the children to the other side?

We noted several responses. Some students (like Kenneth) took a step-by-step approach; they drew each trip back and forth and then counted by 2s. Some students (like Robert and Pedro) tried to look behind the problem, searching for tricks and patterns; they saw that 2 out of 18 children got to the other side for every two trips across the river and understood that they could subtract one trip because the adult didn’t need to make the last trip back. Other students (like Omar) invested more time in chatting with the interviewer and solved the problem more easily by connecting it to some aspect of everyday life, such as unloading groceries from a car. Finally, some students (like Kenneth, in his later attempt) relied more on visualization, using a variety of strategies to explore the situation; these students closed their eyes, drew the situation, or turned it into a story.

All the students we interviewed approached mathematical learning through one or more styles of mathematical thinking. Students with similar learning styles tend to favor specific instructional approaches, struggle with similar sets of concepts, and define mathematics in similar ways (Thomas, 2003b). For example,

- Students favoring the Mastery style learn most easily from teaching approaches that emphasize step-by-step demonstrations and repetitive practice. Students in this group struggle with abstractions, explanations, and non-routine problem solving. They define mathematics as proficiency in calculation and computation.
- Students favoring the Understanding style learn most easily from teaching approaches that emphasize concepts and the reasoning behind mathematical operations. These students struggle with work that emphasizes collaboration, application, and routine drill and practice. They define mathematics primarily in terms of explanations, reasons, and proofs.
- Students favoring the Interpersonal style learn most easily from teaching approaches that emphasize cooperative learning, real-life contexts, and connections to everyday life. Students in this group struggle with independent seatwork, abstraction, and out-of-context, nonroutine problem solving. They define mathematics primarily in terms of applications to everyday life.
- Students favoring the Self-Expressive style learn most easily from teaching approaches that emphasize visualization and exploration. These students struggle with step-by-step computation and routine drill and practice. They define mathematics primarily in terms of nonroutine problem solving.

These different mathematical learning styles provide a map of cognitive diversity among mathematics students. Understanding these styles helps teachers address student strengths and weaknesses as learners. If teachers incorporate all four styles into a math unit, they will build in computation skills.
(Mastery), explanations and proofs (Understanding), collaboration and real-world application (Interpersonal), and nonroutine problem solving (Self-Expressive).

Barb Heinzman, a 5th grade teacher, describes the connection between style-based differentiation and curriculum design for mathematics in this way:

What I saw right away was that not only did different students approach mathematics using different learning styles, but real mathematical power required using all four styles. Think about it: If you can't compute accurately, explain your ideas, discover solutions, and apply math in the real world—you don't know math. Miss even one of these and you miss the boat. The problem with most math programs is they emphasize just one of these and leave out the rest. By building every unit so it includes all four styles of learning, I support all my students, and I stretch them into areas where they wouldn't naturally go. (cited in Strong, Silver, & Perini, 2001)

In other words, any sufficiently important mathematical topic requires students to learn that topic in four dimensions: procedurally, conceptually, contextually, and investigatively.

Still, the question remained how teachers were going to focus on the four dimensions of mathematical learning while accommodating and expanding the different mathematical learning styles of their students.

**The Style-Strategy Connection**

Nearly everyone in education today recognizes the contribution that Robert Marzano, Debra Pickering, and Jane Pollock have made to our understanding of how effective teaching influences student achievement. They have provided an invaluable blueprint for improving instruction and learning by enumerating nine effective teaching strategies in order of their effect sizes. These strategies consist of identifying similarities and differences; summarizing and notetaking; reinforcing effort and providing recognition; homework and practice; nonlinguistic representation; cooperative learning; goal setting and

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**FIGURE 1 Three Sample Beginnings for Learning About Division**

**Version 1: Classroom Strategies That Work Approach**

As a result of this unit, I will be able to:
- Use what I have learned about division to make calculations quickly and accurately.
- Explain how and why I solve division problems the way I do.
- Understand key terms: divide, divisor, dividend, quotient, remainder, fraction, decimal.
- Use problem-solving strategies to solve challenging word problems about division.
- Recognize when division is a useful strategy and apply it to everyday situations.

Personal goals during this unit: I will . . .

(Adapted from Marzano, Pickering, & Pollock, 2001)

**Version 2: Differentiated Instruction Approach**

During our study of division, we want to provide opportunities for you to learn about the topic in ways that work best for you. Number your choices from 1 to 6 (1 = easiest, 6 = most difficult).

- using manipulatives
- observing demonstrations
- sketching out the math situation
- reading
- comparing your work with a partner
- solving complex problems in a team

What do you already know about division?
1. What does "division" mean?
2. How would you explain doing division to someone in a younger grade?
3. What are some examples of division that you know how to do?
4. Where do people use division? And why?

(Adapted from Tomlinson, 2001)

**Version 3: Combined Approach**

In this unit, you will learn:
- How to use division to calculate quickly and accurately.
- How to connect and apply division to everyday situations.
- How to develop good explanations about how and why division works.
- How to use problem-solving strategies, especially diagrams, to solve challenging word problems.

What's your interest?
We will be investigating a number of situations in which division is applied to situations in the real world. Rank these in order of interest, from 1 (most) to 4 (least): sharing food fairly; figuring out how fast a car is going; making money; spending money.

What do you already know about division?
1. What does division mean?
2. Can you give an example of divisor, dividend, quotient, and remainder?
3. What is division for? Why do we need it?
4. What are some examples of division that you know how to do?
5. What makes division difficult?
Feedback; generating and testing hypotheses; and cues, questions, and advance organizers.

But how are these strategies related to the four dimensions of mathematical learning and to students' individual learning styles? Strategies are plans for leading a group or individual to a goal, and they achieve their ends by evoking or scaffolding a particular style of thinking. Thus, strategies, like students, have different styles.

Identifying similarities and differences, for example, stimulates an understanding style of thought that is particularly helpful in equipping students to develop sound explanations for the reasons underlying mathematical operations. Advance organizers, on the other hand, impose an organizational structure that can be particularly helpful in developing students' capacities to learn and articulate particular computational procedures.

Teachers can use such strategies to differentiate student learning in a number of ways. They can

- **Rotate strategies**: Identify learning goals, then deliberately use multiple strategies over the course of a unit to guarantee that all students believe that the variety of strategies both validates their dominant style and challenges them to work in less preferred styles.

- **Use flexible grouping**: Identify a common purpose, such as developing accurate explanations in a unit on time, rate, and distance problems, and then divide students into different style groups that use alternate strategies. Style groups can validate or challenge students' dominant styles, depending on whether groups are style-alike or style-diverse.

- **Personalize learning**: When students struggle or need an extra challenge, shift the strategy to individualize their instruction. For example, a middle school math student with a penchant for creativity and imaginative exercises was having trouble internalizing the steps in the operation-solving process. His teacher then used a Self-Expressive strategy to challenge him to create a metaphor—he chose digestion—for the operation-
solving process.

Making the style-strategy connection helps teachers maintain a unity of focus across a variety of mathematical topics while simultaneously differentiating to meet the needs of learners' different styles. But can teachers sustain this sense of balance when it comes to assessing student progress?

**Fair and Thoughtful Tests**

We have seen that although every student is unique, student differences in learning mathematics tend to cluster into four mathematical learning styles. We have also seen that each of these styles tends toward one of the four dimensions of mathematical learning, either computation, explanation, application, or problem solving. Finally, we have seen that teachers can categorize research-based teaching strategies according to both the mathematical capacities they support and the learning styles they evoke and scaffold.

When we turn this lens on the textbooks that teachers use and the tests that those textbooks supply, however, we find that both the tests and their tests tend to emphasize one or two styles of mathematical learning and assessment—often Mastery and Understanding—and exclude or marginalize the others—usually Self-Expressive and Interpersonal. If students possess different mathematical learning styles, and if attaining full mathematical power requires all four dimensions of mathematics, then these tests are neither fair nor balanced. We should supplement or replace them with assessments that do justice to both our standards and our students. We can easily imagine such an assessment on student understanding of fractions. At the top of the page, the standards would be addressed. Below that, a four-part grid would address each dimension of mathematical learning and all four mathematical learning styles by asking students to compute with fractions, explain how fractions work, apply fractions to a real-life situation, and solve a nonroutine problem using fractions.
Constructing a Differentiated Mathematics Classroom

In her address at the 2003 ASCD Annual Conference, Tomlinson (2003) reminded us that differentiation does not depend on simply using this or that strategy. Certain principles of quality, she explained, matter more: a commitment to growth, helping students see themselves in a better light; a belief in challenging all students and giving them a feeling of accomplishment; a sense of community, where every student knows that he or she has something special to contribute; and a determination to create a solid curriculum, one informed by a clear sense of purpose, a magnet-like relationship to assessment, and a belief that curriculum should offer all students “a way up, not a way out.”

Mathematics often seems like a kind of worst-case scenario for differentiation. Both the nature of its content and its largely sequential organization make considerations of student differences seem peripheral. We can, however, begin to construct the kind of mathematics classrooms that Tomlinson has helped us imagine if we can make a commitment to

- Include all four dimensions of mathematical learning—computation, explanation, application, and problem solving—in every unit we teach;
- Help students recognize their own mathematical learning styles—Mastery, Understanding, Interpersonal, or Self-Expressive—along with their strengths, their weaknesses, and where they need to grow;
- Use a variety of teaching strategies to explore mathematical topics; and
- Create or revise our assessments to reflect all four dimensions of mathematical learning and all four learning styles that students use to approach those dimensions.

Perhaps mathematics can then take the lead in showing other disciplines how to combine unity of focus with thoughtful differentiation to create a new world of student achievement. It’s a good place to start.

References


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